

## 5.8 | Relativistic Momentum

### Learning Objectives

By the end of this section, you will be able to:

- Define relativistic momentum in terms of mass and velocity
- Show how relativistic momentum relates to classical momentum
- Show how conservation of relativistic momentum limits objects with mass to speeds less than  $c$

Momentum is a central concept in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions (**Figure 5.23**). Much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles, and momentum conservation plays a crucial role in this analysis.



**Figure 5.23** Momentum is an important concept for these football players from the University of California at Berkeley and the University of California at Davis. A player with the same velocity but greater mass collides with greater impact because his momentum is greater. For objects moving at relativistic speeds, the effect is even greater.

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? It can be shown that the momentum calculated as merely

$\vec{p} = m \frac{d\vec{x}}{dt}$ , even if it is conserved in one frame of reference, may not be conserved in another after applying the

Lorentz transformation to the velocities. The correct equation for momentum can be shown, instead, to be the classical expression in terms of the increment  $d\tau$  of proper time of the particle, observed in the particle's rest frame:

$$\begin{aligned}
 \vec{p} &= m \frac{d\vec{x}}{d\tau} = m \frac{d\vec{x}}{dt} \frac{dt}{d\tau} \\
 &= m \frac{d\vec{x}}{dt} \frac{1}{\sqrt{1 - u^2/c^2}} \\
 &= \frac{m \vec{u}}{\sqrt{1 - u^2/c^2}} = \gamma m \vec{u}.
 \end{aligned}$$

### Relativistic Momentum

**Relativistic momentum**  $\vec{p}$  is classical momentum multiplied by the relativistic factor  $\gamma$ :

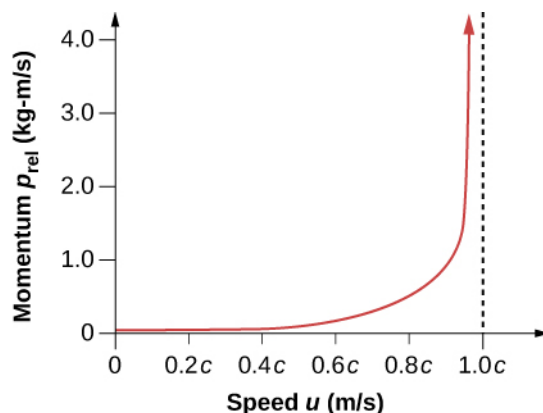
$$\vec{p} = \gamma m \vec{u} \quad (5.6)$$

where  $m$  is the **rest mass** of the object,  $\vec{u}$  is its velocity relative to an observer, and  $\gamma$  is the relativistic factor:

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}. \quad (5.7)$$

Note that we use  $u$  for velocity here to distinguish it from relative velocity  $v$  between observers. The factor  $\gamma$  that occurs here has the same form as the previous relativistic factor  $\gamma$  except that it is now in terms of the velocity of the particle  $u$  instead of the relative velocity  $v$  of two frames of reference.

With  $p$  expressed in this way, total momentum  $p_{\text{tot}}$  is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical quantity at low velocities, where  $u/c$  is small and  $\gamma$  is very nearly equal to 1. Relativistic momentum has the same intuitive role as classical momentum. It is greatest for large masses moving at high velocities, but because of the factor  $\gamma$ , relativistic momentum approaches infinity as  $u$  approaches  $c$  (Figure 5.24). This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite—an unreasonable value.



**Figure 5.24** Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

The relativistically correct definition of momentum as  $p = \gamma mu$  is sometimes taken to imply that mass varies with velocity:  $m_{\text{var}} = \gamma m$ , particularly in older textbooks. However, note that  $m$  is the mass of the object as measured by a person at rest relative to the object. Thus,  $m$  is defined to be the rest mass, which could be measured at rest, perhaps using gravity. When a mass is moving relative to an observer, the only way that its mass can be determined is through collisions or other means involving momentum. Because the mass of a moving object cannot be determined independently of momentum, the only meaningful mass is rest mass. Therefore, when we use the term “mass,” assume it to be identical to “rest mass.”

Relativistic momentum is defined in such a way that conservation of momentum holds in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This

has been verified in numerous experiments.



**5.8 Check Your Understanding** What is the momentum of an electron traveling at a speed  $0.985c$ ? The rest mass of the electron is  $9.11 \times 10^{-31}$  kg.

## 5.9 | Relativistic Energy

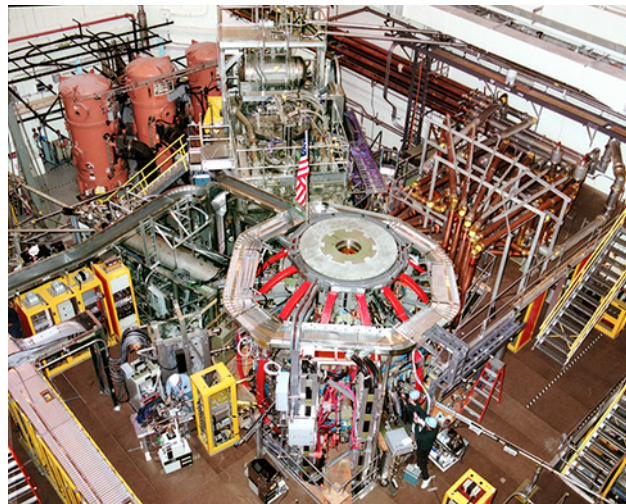
### Learning Objectives

By the end of this section, you will be able to:

- Explain how the work-energy theorem leads to an expression for the relativistic kinetic energy of an object
- Show how the relativistic energy relates to the classical kinetic energy, and sets a limit on the speed of any object with mass
- Describe how the total energy of a particle is related to its mass and velocity
- Explain how relativity relates to energy-mass equivalence, and some of the practical implications of energy-mass equivalence

The tokamak in **Figure 5.25** is a form of experimental fusion reactor, which can change mass to energy. Nuclear reactors are proof of the relationship between energy and matter.

Conservation of energy is one of the most important laws in physics. Not only does energy have many important forms, but each form can be converted to any other. We know that classically, the total amount of energy in a system remains constant. Relativistically, energy is still conserved, but energy-mass equivalence must now be taken into account, for example, in the reactions that occur within a nuclear reactor. Relativistic energy is intentionally defined so that it is conserved in all inertial frames, just as is the case for relativistic momentum. As a consequence, several fundamental quantities are related in ways not known in classical physics. All of these relationships have been verified by experimental results and have fundamental consequences. The altered definition of energy contains some of the most fundamental and spectacular new insights into nature in recent history.



**Figure 5.25** The National Spherical Torus Experiment (NSTX) is a fusion reactor in which hydrogen isotopes undergo fusion to produce helium. In this process, a relatively small mass of fuel is converted into a large amount of energy. (credit: Princeton Plasma Physics Laboratory)

### Kinetic Energy and the Ultimate Speed Limit

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy of a particle is valid relativistically, but for energy expressed in terms of velocity and mass in